

20th International Summer School

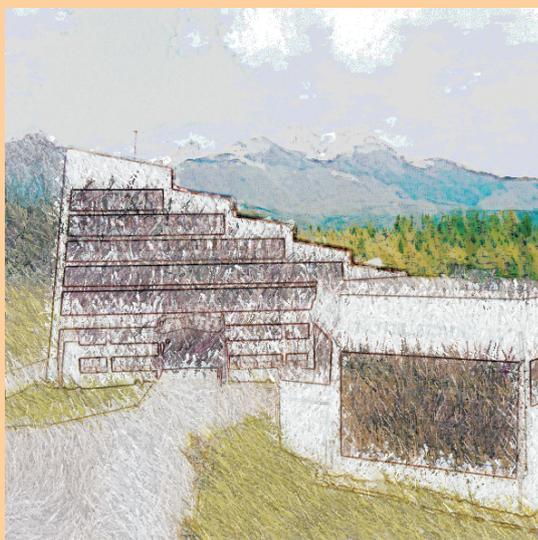
on

Global Analysis
and its **A**pplications

(extended abstract book)

August 17 - 21, 2015

Stará Lesná



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SLOVAKIA

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20th International Summer School

on

Global Analysis and its Applications
EXTENDED ABSTRACT BOOK

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EXTENDED ABSTRACT BOOK

20th International Summer School on Global Analysis and its Applications
August 17-21, 2015, Stará Lesná, Slovakia

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Preface

The topic of the 20th International Summer School on Global Analysis and its Applications was "General Relativity: 100 years after Hilbert" and was devoted to the 100th anniversary of the general relativity theory. It was aimed as a contribution to numerous international scientific events, related to the founders of the theory, Albert Einstein and David Hilbert. The event was held in Academia Congress Center situated in the village of Stará Lesná in the beautiful area of the Slovak mountains, the High Tatras.

The research program of the meeting consisted of several parts. First of all, renowned specialists in the field, Professor Salvatore Capozziello (Università Di Napoli "Federico II", Italy) and Professor Demeter Krupka (Lepage Research Institute and University of Hradec Králové) presented two series of lectures with titles "Mathematical Foundations of Metric-Affine Theories of Gravity" and "The Variational Foundations of General Relativity Theory", respectively. Secondly, eight well-known invited speakers, Professors Vladimir Chernov (USA), Marco Ferraris (Italy), Jerzy Kijowski (Poland), Marcella Palese (Italy), Istvan Racz (Hungary), Miguel Sánchez (Spain), Gennady Sardanashvily (Russia) and Nicoleta Voicu (Romania) delivered the lectures on results concerning the general relativity theory. The third part included short communications and posters of the participants on their current research results.

High-quality scientific program and lecturers attracted finally 64 participants from 24 countries to attend the School. From this point of view this year's Summer School has become the most successful among all 20 editions. For the organizers, I would like to thank all participants for their contribution to the program either by nice prepared and delivered lectures or by discussions on different research topics. The organization of the 2015 Summer School was supported by the International Visegrad Fund. We also appreciate the help of the main organizers, Lepage Research Institute (Czech Republic), and University of Prešov (Slovakia) - also for the financial support provided by its Faculty of Humanities and Natural Sciences, and the support of other organizers, Jagiellonian University in Kraków (Poland), Eötvös Loránd University in Budapest (Hungary) and University of Hradec Králové (Czech Republic) in the field of promotion of the Summer School in the corresponding countries. We invite all of you to attend next summer schools in the series.

Stará Lesná, 21 August 2015

Ján Brajerčík

On behalf of the Organizing Committee

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VARIATIONAL FOUNDATIONS OF GENERAL RELATIVITY THEORY

Demeter Krupka

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ABSTRACT

Contents of lectures

1. Variational principles on fibered manifolds

Jets of differentiable mappings, Lagrange structures, first variation, Euler-Lagrange form, invariant variational functionals, the inverse problem and variational completion, energy-momentum tensors

2. Differential invariants and lifting functors

Differential groups, differential invariants, examples, frames, liftings in principal and associated bundles

3. Natural Lagrange structures

Liftings of diffeomorphisms and vector fields, natural Lagrangians, natural source forms, Euler-Lagrange forms and field equations, conservation laws, Noether's theorems, examples

4. Natural variational principles for the metric fields

The Hilbert variational functional, natural Lagrangians depending on tensors, examples, invariance: conservation laws, Noether's theorems

LINKING, CAUSALITY AND SMOOTH STRUCTURES ON SPACETIMES**Vladimir Chernov**

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ABSTRACT

Globally hyperbolic spacetimes form probably the most important class of spacetimes. Low conjecture and the Legendrian Low conjecture formulated by Natário and Tod say that for many globally hyperbolic spacetimes X two events x, y in X are causally related if and only if the link of spheres S_x, S_y whose points are light rays passing through x and y is non-trivial in the contact manifold N of all light rays in X . This means that the causal relation between events can be reconstructed from the intersection of the light cones with a Cauchy surface of the spacetime.

We prove the Low and the Legendrian Low conjectures and show that similar statements are in fact true in almost all 4-dimensional globally hyperbolic spacetimes. This also answers the question on Arnold's problem list communicated by Penrose.

We also show that on many 4-manifolds there is a unique smooth structure underlying a globally hyperbolic Lorentz metric, thus global hyperbolicity imposes censorship on the possible smooth structures on a spacetime. For instance, every contractible smooth 4-manifold admitting a globally hyperbolic Lorentz metric is diffeomorphic to the standard R^4 .

INVARIANT VARIATIONAL PROBLEMS AND COHOMOLOGY

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ABSTRACT

We will pose the inverse problem question within the Krupka's variational sequence framework. In particular, the interplay of inverse problems with symmetry and invariance properties will be exploited considering that the cohomology class of the variational Lie derivative of an equivalence class of forms, closed in the variational sequence, is trivial. We will focalize on the case of symmetries of globally defined field equations which are only locally variational and find sufficient conditions for the variation of local Noether strong currents to be conserved and variationally equivalent to a global conserved current.

THE MANY FACES OF THE CONSTRAINTS IN GENERAL RELATIVITY

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ABSTRACT

In this talk the constraint equations for $[n+1]$ -dimensional (with $n > 3$) smooth Riemannian and Lorentzian spaces satisfying Einstein's equations will be considered. Under some mild topological assumptions it is shown first that whenever the primary space is Riemannian the 'Hamiltonian' and 'momentum' type expressions satisfy exactly the same type of first order symmetric hyperbolic subsidiary system as they do in the conventional Lorentzian case. It is shown then that, regardless whether the primary space is Riemannian or Lorentzian, the constraints can always be put into the form of an evolutionary system comprised either by a first order symmetric hyperbolic system and a parabolic equation or, alternatively, by a strongly hyperbolic system subsided by an algebraic relation. The (local) existence and uniqueness of solutions to these evolutionary systems is also shown verifying thereby that the proposed evolutionary approach provides a viable alternative to the apparently unique conformal method.

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MSC2010: 83C05, 83C05, 53Z05

Keywords: non-linear dynamical systems, initial value problem, evolutionary systems, constraints, Riemannian and Lorentzian spaces, foliations.

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The author is grateful to the Albert Einstein Institute in Golm, Germany for its kind hospitality where parts of the reported results were derived. This research was also supported in part by the Die Aktion Österreich-Ungarn, Wissenschafts- und Erziehungskooperation grant 90öu1.

SOME LINKS BETWEEN LORENTZIAN AND FINSLERIAN GEOMETRIES

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ABSTRACT

There are two different natural links between Lorentzian and Finslerian geometries.

On the one hand, the causality of spacetimes can be computed from a Finslerian metric on its spacelike slices. This yields a purely geometric correspondence between relativistic elements (causality, gravitational lensing, causal boundaries) and Finslerian ones (non-symmetric distances, convexity and Busemann boundaries, resp.) Its applications include even the full description of non-relativistic Zermelo's navigation, i.e., to find the fastest trajectory between two points under a (possibly strong) wind.

On the other, some authors have considered the possibility to extend classical General Relativity to a Finsler-Lorentz setting, obtaining so a theory of modified gravity whose applications would include models of dark energy and quantum effects. This puts forward the development of a global Lorentz-Finsler Geometry.

A summary of these topics and prospective questions will be explained along the talk.

GAUGE GRAVITATION THEORY. GRAVITY AS A HIGGS FIELD

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ABSTRACT

Classical gravitation theory on a world manifold X is formulated as gauge theory on natural bundles over X which admit general covariant transformations as the canonical functorial lift of diffeomorphisms of their base X . Natural bundles are exemplified by a principal linear frame bundle $LX \rightarrow X$ and the associated, (e.g., tensor) bundles. This is metric-affine gravitation theory whose dynamic variables are general linear connections (principal connections on LX) and a metric (tetrad) gravitational field. The latter is represented by a global section of the quotient bundle $\Sigma = LX/L$ and, thus, it is treated as a classical Higgs field responsible for the reduction of a structure group $GL(4, \mathbb{R})$ of LX to a Lorentz group $SO(1, 3)$. The underlying physical reason of this reduction is both the geometric Equivalence Principle and the existence of Dirac spinor fields. Herewith, a structure Lorentz group of LX always is reducible to its maximal compact subgroup $SO(3)$ that provides a world manifold X with a space-time structure. The physical nature of gravity as a Higgs field is characterized by the fact that, given different tetrad gravitational fields h , the representations $dx^\mu \mapsto h_a^\mu \gamma^a$ of holonomic coframes $\{dx^\mu\}$ on a world manifold X by Dirac's γ -matrices are non-equivalent. Consequently, the Dirac operators in the presence of different gravitational fields fails to be equivalent, too. To solve this problem, we describe Dirac spinor fields in terms of a composite spinor bundle $S \rightarrow \Sigma \rightarrow X$ where $S \rightarrow \Sigma$ is a spinor bundle associated with a $SO(1, 3)$ -principal bundle $LX \rightarrow \Sigma$. A key point is that, given a global section h of $\Sigma \rightarrow X$, the pull-back bundle h^*S of $S \rightarrow \Sigma$ describes Dirac spinor fields in the presence of a gravitational field h . At the same time, $S \rightarrow X$ is a natural bundle which admits general covariant transformations. As a result, we obtain a total Lagrangian of a metric-affine gravity and Dirac spinor fields, whose gauge invariance under general covariant transformations implies an energy-momentum conservation law. Our physical conjecture is that a metric gravitational field as the Higgs one is non-quantized, but it is classical in principle.

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MSC2010: 53B50, 70S15, 83D05

Keywords: gauge theory, gravitation theory, Higgs field, natural bundle, principal connection, reduced structure.

ENERGY-MOMENTUM TENSOR AND VARIATIONAL COMPLETION**Nicoleta Voicu**

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ABSTRACT

The idea of finding the energy-momentum tensor of a classical field theory as a kind of „Noether current” associated to the invariance of the corresponding matter Lagrangian to some (or even the entire) group of spacetime diffeomorphisms is already many decades old and it was, for sure, a subject of numerous debates.

The majority of the approaches start from the canonical, or Noether energy-momentum tensor - or from a covariant version of it – and try to „improve” it, by adding correction terms; this way, one finds, in the particular case of general relativity, the Hilbert energy-momentum tensor as a consequence. In this respect, a remarkable paper is the one by Gotay and Marsden (2001), which presents a geometric approach to the problem, on general fibered manifolds.

After a brief review of the problems surrounding energy-momentum tensors and of the Gotay-Marsden improvement procedure (with some clarifications by Forger and Romer, 2004), we redo the whole construction the other way around. Namely, we define energy-momentum tensors via a generalized Hilbert-type procedure and find out that they give the correct Noether currents as a consequence. This not only simplifies the computations and the proofs of the corresponding results, but also opens up new possibilities. One of these possibilities is the use of the technique of variational completions for finding the full expression of an energy-momentum tensor in the case when we know by some means a piece of it – and even finding a Lagrangian for the theory, in the case when this is not known.

Keywords: Lepage equivalent of a Lagrangian, inverse variational problem, source form, variational completion.

**OBTAINING GENERAL RELATIVITY'S N-BODY NON-LINEAR
LAGRANGIAN FROM ITERATIVE,
LINEAR ALGEBRAIC SCALING EQUATIONS**

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ABSTRACT

General Relativity's gravity produces from a matter distribution both a set of non-linear metric field potentials $g_{\mu\nu}$ establishing the space-time arena and its local proper coordinates for all other physics, and also the non-linear gravitational equations of motion for the matter in that arena. Several invariance properties of General Relativity's gravity — **exterior effacement, interior effacement, and the time dilation and Lorentz contraction features of matter in Minkowski space** — are here exploited to develop an iterative, linear algebraic method for obtaining the N-body non-linear gravitational Lagrangian to all orders (excluding gravitational radiation reaction effects, but see [1, 2].)

Exterior effacement means that the gravitational Lagrangian for a general local N-body system of bodies is not altered by distant distributions of spectator bodies when the local system Lagrangian is expressed in local proper space-time coordinates produced by the spectator bodies. For creating necessary (but not sufficient) conditions for achieving exterior effacement, consideration can be restricted to the motion-independent potentials for a general N-body system of bodies surrounded by Minkowski space; see [1]. The temporal metric field potential g_{00} , the isotropic part of the spatial metric field potential $-g_{ab} \sim \delta_{ab}$, and the N-body Lagrangian include non-linear infinite-order expansions of motion-independent potentials of the form:

$$\begin{aligned} -g_{ab}(\vec{r}) &= \left(1 + \sum_{n,\alpha} \kappa(n,\alpha) V(n,\alpha,\vec{r}) \right) \delta_{ab} + \dots \\ &= \left(1 + 2 \sum_i \frac{G m_i}{c^2 r_i} + \kappa(2,1) \sum_{ij} \frac{G^2 m_i m_j}{c^4 r_i r_j} + \kappa(2,2) \sum_{ij} \frac{G^2 m_i m_j}{c^4 r_i r_{ij}} + \dots \right) \delta_{ab} + \dots \end{aligned} \quad (1)$$

$$\begin{aligned} g_{00}(\vec{r}) &= 1 + \sum_{n,\alpha} \xi(n,\alpha) V(n,\alpha,\vec{r}) + \dots \\ &= 1 - 2 \sum_i \frac{G m_i}{c^2 r_i} + \xi(2,1) \sum_{ij} \frac{G^2 m_i m_j}{c^4 r_i r_j} + \xi(2,2) \sum_{ij} \frac{G^2 m_i m_j}{c^4 r_i r_{ij}} + \dots \end{aligned} \quad (2)$$

$$L = \sum_{n,\epsilon} \lambda(n, \epsilon) U(n, \epsilon, r_{ij}, r_{jk}, \dots) + \dots = \frac{1}{2} \sum_{ij} \frac{G m_i m_j}{r_{ij}} + \lambda(2, 1) \sum_{ijk} \frac{G^2 m_i m_j m_k}{c^2 r_{ij} r_{ik}} + \dots \quad (3)$$

with $r_i = |\vec{r} - \vec{r}_i|$, $r_{ij} = |\vec{r}_j - \vec{r}_i|$, etc. The $+\dots$ are acknowledgments that the full metric field and Lagrangian expansions include motion-dependent potentials as well. Note that the potentials V in the metric field component expansions and potentials U in the N-body Lagrangian expansion are different; the former have a field point \vec{r} and latter contain only interbody distances r_{ij} . The α and ϵ indices run over the number of distinct potential types appearing at each order in $Gm/c^2 r$.¹ Beginning with known lowest order terms for the three infinite series of potentials, Equations 1-3, the goal is to develop an iterative process which determines by linear algebraic equations the coefficients $\kappa(n, \alpha)$, $\xi(n, \alpha)$, and $\lambda(n, \epsilon)$ in terms of the known lower order $n' < n$ coefficients so as to enforce exterior effacement and other properties of gravity. Equations 1-3 are assumed to apply to a general system of bodies consisting of distant spectator bodies $s, s', s'' \dots$ at large distances $R_s, R_{s'} \dots$ from a localized system of bodies $i, j, k \dots$ in whose vicinity the spectator bodies produce rescaled space and time proper coordinates. If the metric potentials in the expansions of Equations 1-2 are confined to the spectator body sources located at great distances R_s from the local system and separations $R_{ss'}$ from each other — for $i, j \rightarrow s, s' \dots r_i \rightarrow R_s, r_{ij} \rightarrow R_{ss'}, \text{ etc.}$, $V(n, \alpha, \vec{r}) \rightarrow \bar{V}(n, \alpha)$, the local metric field space and time scaling factors are produced:

$$-g_{ab}(\vec{r}) \rightarrow -\bar{g}_{SS} \delta_{ab} = \left(1 + \sum_{n,\alpha} \kappa(n, \alpha) \bar{V}(n, \alpha) \right) \delta_{ab}$$

$$g_{00}(\vec{r}) \rightarrow \bar{g}_{00} = 1 + \sum_{n,\alpha} \xi(n, \alpha) \bar{V}(n, \alpha)$$

giving the proper coordinates for the local system; $(\bar{g}_{00})^{1/2} dt = d\tau$ and $(-\bar{g}_{SS})^{1/2} d\vec{r} = d\vec{\rho}$.

As example of exterior effacement, its lowest order manifestation can be exhibited by dividing the Newtonian potential summation into its local body contributions and its contribution from all the spectator bodies, s , $U = \sum Gm_s/c^2 R_s$, and so to linear order $(1 - U)dt \approx d\tau$, $(1 + U)dr \approx d\rho$:

$$\sum_i \frac{G m_i}{c^2 r_i} \rightarrow \sum_{\text{local } i} \frac{G m_i}{c^2 r_i} + U \quad \sum_{ij} \frac{G m_i m_j}{c^2 r_{ij}} \rightarrow 2U \sum_{\text{local } i} m_i$$

$$\sum_{ijk} \frac{G^2 m_i m_j m_k}{c^2 r_{ij} r_{ik}} \rightarrow 2U \sum_{\text{local } ij} \frac{G m_i m_j}{r_{ij}}$$

and then the lowest order scalings of the Lagrangian expansion given by Equation 3 (plus

¹These α range numbers grow rapidly with order n , being 1, 2, 4, 9, 20, 48, 115, 286, respectively, for $n = 1$ to 8, for example, and similar escalation for the ϵ range numbers.

the body rest energies Lagrangian) produces

$$\left(- \sum_{\text{local } i} m_i c^2 + L \right) dt \longrightarrow \left((-1 + U) \sum_{\text{local } i} m_i c^2 + \left(\frac{1}{2} (1 + U) + 2 \lambda(2, 1) U \right) \sum_{\text{local } ij} \frac{G m_i m_j}{\rho_{ij}} \right) dt \quad (4)$$

For $\lambda(2, 1) = -1/2$ an overall $1 - U$ factors out and combines with dt to produce the local proper $d\tau$, and the spectator bodies are effaced to linear order. Newton's G is locally unaffected by distribution of distant matter.

The potentials appearing in Equations 1-2, by various specified partitions of the body summations into spectator bodies and local bodies, become spectator scaling factors times lower order local system N-body potentials, and this split of any n th order potential generally exists in multiple ways. For example, from among the nine fourth order potentials ² the complete set of partitionings of $V(4, 4, \vec{r})$ is found to be

$$\begin{aligned} V(4, 4, \vec{r}) = \sum_{i,j,k,l} \frac{G^4 m_i m_j m_k m_l}{c^8 r_i r_j r_k r_l} \longrightarrow & \left[\sum_s \frac{G m_s}{c^2 R_s} \right] \sum_{\text{local } i,j,k} \left(\frac{G^3 m_i m_j m_k}{c^6 r_i r_j r_k} + 2 \frac{G^3 m_i m_j m_k}{c^6 r_i r_j r_{jk}} \right) \\ & + \left[\sum_s \frac{G m_s}{c^2 R_s} \right]^2 \sum_{\text{local } i,j} \left(\frac{G^2 m_i m_j}{c^4 r_i r_j} + 2 \frac{G^2 m_i m_j}{c^4 r_i r_{ij}} \right) \\ & + \left[\left(\sum_s \frac{G m_s}{c^2 R_s} \right)^3 + \sum_{ss's''} \frac{G^3 m_s m_{s'} m_{s''}}{c^6 R_s R_{s's''} R_{ss''}} \right] \sum_{\text{local } i} \frac{G m_i}{c^2 r_i} \end{aligned}$$

with the various spectator potential scale factors shown in [...] brackets. But external effacement requires that every local system potential must receive from all higher order potentials a specific total rescaling. For the potentials appearing in the spatial and temporal metric components the total scaling factor of an n th order potential with n powers of $1/r$ must be $(-\bar{g}_{SS})^{1-n/2}$ and $(\bar{g}_{00})^1 (-\bar{g}_{SS})^{-n/2}$, respectively, so that

$$(-\bar{g}_{SS})^{1-n/2} V(n, \alpha, \vec{r}) dr^2 \longrightarrow V(n, \alpha, \vec{\rho}) d\rho^2$$

and

$$(\bar{g}_{00})^1 (-\bar{g}_{SS})^{-n/2} V(n, \alpha, \vec{r}) dt^2 \longrightarrow V(n, \alpha, \vec{\rho}) d\tau^2$$

and the metric potential expansions reproduce themselves identically when expressed in the new local proper coordinates. Requiring this to all orders in spectator scaling then yields the iterative linear algebraic equations for the expansion coefficients $\kappa(n, \alpha)$ and $\xi(n, \alpha)$

$$\sum_{\alpha} c(n, \alpha; n', \alpha', \beta) \kappa(n, \alpha) = \left(\frac{\partial}{\partial \bar{V}(n - n', \beta)} (-\bar{g}_{SS})^{1-n'/2} \right) \kappa(n', \alpha') \quad (5)$$

²The nine $V(4, \alpha, \vec{r})$ potentials range from the Schwarzschild $V(4, 1, \vec{r})$ proportional to $m_i m_j m_k m_l / r_i r_j r_k r_l$ to $V(4, 9, \vec{r})$ proportional to $m_i m_j m_k m_l / r_i r_{ij} r_{jk} r_{kl}$

$$\sum_{\alpha} c(n, \alpha; n', \alpha', \beta) \xi(n, \alpha) = \left(\frac{\partial}{\partial \bar{V}(n - n', \beta)} (\bar{g}_{00})^1 (-\bar{g}_{SS})^{-n'/2} \right) \xi(n', \alpha') \quad (6)$$

with there being a different linear equation for each combination of $n' < n$, α' , and β . The numbers $c(n, \alpha; n', \alpha', \beta)$ are simply the integer counts of how many ways (including possibly zero) the potential $V(n, \alpha, \vec{r})$ factors into the lower order potential $V(n', \alpha', \vec{r})$ times spectator potential $\bar{V}(n - n', \beta)$. The partial derivatives of the required total scaling factors on right hand side of Equations 5,6 simply pull out of those total scaling factors the numerical coefficients of the specific indicated spectator potentials.

Once the spatial and temporal metric field components are determined, the iterative algebraic equations for the motion-independent Lagrangian potentials can be constructed. The Lagrangian potentials' total scaling factors required for exterior effacement are

$$(\bar{g}_{00})^{1/2} (-\bar{g}_{SS})^{-n'/2}$$

so that

$$(\bar{g}_{00})^{1/2} (-\bar{g}_{SS})^{-n'/2} U(n, \epsilon, r_{ij}, r_{jk} \dots) dt \longrightarrow U(n, \epsilon, \rho_{ij}, \rho_{jk} \dots) d\tau \quad (7)$$

The resulting set of iterative linear algebraic equations for the Lagrangian coefficients $\lambda(n, \epsilon)$ are then

$$\sum_{\epsilon} c^*(n, \epsilon; n', \epsilon', \beta) \lambda(n, \epsilon) = \left(\frac{\partial}{\partial \bar{V}(n - n', \beta)} (\bar{g}_{00})^{1/2} (-\bar{g}_{SS})^{-n'/2} \right) \lambda(n', \epsilon') \quad (8)$$

for each $n' < n$, each ϵ' , and each β . The numbers $c^*(n, \epsilon; n', \epsilon', \beta)$ are now the integer counts of how many ways (including possibly zero) the Lagrangian potential $U(n, \epsilon, r_{ij}, r_{jk} \dots)$ factors into the lower order potential $U(n', \epsilon', r_{ij}, r_{jk} \dots)$ times spectator potential $\bar{V}(n - n', \beta)$.

Time dilation invariance means that any motions of a gravitational dynamical system must also be possible with respect to the dilated time variable of special relativity if the dynamical system is viewed from a moving frame of reference. If a planar but otherwise general N-body system is given a boost \vec{w} perpendicular to its planar dynamics, its kinetic Lagrangian is invariant under transformation to the dilated time variable $d\tau^* = \sqrt{1 - w^2/c^2} dt$. With the planar velocities $\vec{u}_i = d\vec{r}_i/d\tau^*$ being defined in terms of the dilated time, each Lagrangian potential rescaled by a boost perpendicular to planar dynamics must then have the scaling property

$$L(\vec{r}_{ij}, \vec{r}_{ik} \dots, \vec{v}_i + \vec{w} \dots) dt = L(\vec{r}_{ij}, \vec{r}_{ik} \dots, \vec{u}_i \dots) \left(\sqrt{1 - w^2/c^2} dt \right)$$

This requirement produces further iterative linear algebraic equations for the coefficients of the motion-dependent Lagrangian potentials.

Lorentz contraction invariance requires that any static configuration of matter requires the Lorentz-contracted version of that system to be allowed when viewed from another

velocity frame. Consider a configuration of bodies at rest held in a static lattice by a number of interaction types α (including gravity) with potentials $U(r_{ij}, \dots, \alpha)$ Then

$$\sum_{\alpha} \vec{\nabla}_a U(r_{ij}, \dots, \alpha) = 0 \quad \text{for all body sites } \vec{r}_a$$

The same lattice in constant motion \vec{w} has its interbody distances Lorentz contracted such that

$$r'_{ij} = r_{ij} \sqrt{1 - (\vec{w} \cdot \hat{r}_{ij})^2 / c^2} \quad \text{and} \quad (\vec{w} \cdot \hat{r}'_{ij})^2 = (\vec{w} \cdot \hat{r}_{ij})^2 \frac{1 - w^2/c^2}{1 - (\vec{w} \cdot \hat{r}_{ij})^2 / c^2} \quad (9)$$

and each static potential type in the Lagrangian acquires an infinite series of velocity-dependent supplements

$$U(r_{ij}, \dots, \alpha) \longrightarrow U(r'_{ij}, \dots, \alpha) \left(1 + F(w^2/c^2, (\vec{w} \cdot \hat{r}_{ij})^2/c^2, \dots, \alpha) \right)$$

But the bodies in the moving lattice must still be free of acceleration. For this to be true for the general boost, the modified motion-dependent potentials must be free of $\vec{w} \cdot \hat{r}_{ij}$ terms to all orders and for each potential type. For example; the Newtonian potential term in the Lagrangian has its denominators r_{ij} contracted for the boosted pairs according to Equation 9, so the motion-dependent numerator factors must take the form

$$1 + F(w^2/c^2, (\vec{w} \cdot \hat{r}_{ij})^2/c^2, \dots, \alpha = 1) = \sqrt{1 - (\vec{w} \cdot \hat{r}_{ij})^2/c^2} \sqrt{1 - w^2/c^2}.$$

Imposing these conditions to each of the static gravitational potentials then produces iterative linear algebraic conditions ubiquitously on the motion-dependent potentials in the gravitational N-body Lagrangian.

The **Interior Effacement** property of gravity requires that all composite bodies in the spherical limit are characterized by only their single "mass" attributes — total mass-energies — in the N-body Lagrangian. A single composite gaseous body's mass-energy is

$$\begin{aligned} M = & \frac{1}{c^2} \left(\sum_i \vec{v}_i \cdot \frac{\partial L}{\partial \vec{v}_i} - L + \dots \right) = \sum_i m_i \left(1 + \frac{1}{2c^2} \left[u_i^2 - \sum_j \frac{G m_j}{\rho_{ij}} \right] \right. \\ & \left. + \frac{1}{c^4} \left[\frac{3}{8} u_i^4 + \frac{1}{2} \sum_{jk} \frac{G^2 m_j m_k}{\rho_{ij} \rho_{ik}} + \frac{1}{4} \sum_j \frac{G m_j}{\rho_{ij}} (3 u_{ij}^2 - \vec{u}_i \cdot \vec{u}_j - \vec{u}_i \cdot \hat{r}_{ij} \hat{r}_{ij} \cdot \vec{u}_j) \right] \right) + \dots \end{aligned} \quad (10)$$

in which the composite body atoms' variables $\vec{\rho}_i$ and $\vec{u}_i \dots$ are expressed in the proper coordinates rescaled not only by any distant spectator masses but also by any other nearby body potentials. And then the total energy of a collection of composite bodies takes the identical form expressed in terms of the composite body mass parameters M_I given by Equation 10,

their interbody separations R_{IJ} , and velocities \vec{W}_I , etc.

$$\begin{aligned}
 E = & \left(\sum_i \vec{v}_i \cdot \frac{\partial L}{\partial \vec{v}_i} - L + \dots \right) = \sum_I M_I c^2 \left(1 + \frac{1}{2c^2} \left[W_i^2 - \sum_J \frac{G M_J}{R_{IJ}} \right] \right. \\
 & + \frac{1}{c^4} \left[\frac{3}{8} W_I^4 + \frac{1}{2} \sum_{JK} \frac{G^2 M_j M_k}{R_{IJ} R_{IK}} + \frac{1}{4} \sum_J \frac{G M_j}{R_{IJ}} \left(3 W_{IJ}^2 - \vec{W}_I \cdot \vec{W}_J - \vec{W}_I \cdot \hat{R}_{IJ} \hat{R}_{IJ} \cdot \vec{W}_J \right) \right] \left. \right) \\
 & + \dots
 \end{aligned} \tag{11}$$

An important empirical consequence of interior effacement is that a composite, self-gravitating body's gravitational mass equals its inertial mass, as confirmed to good precision by 45 years of lunar laser ranging.

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**A POLYNOMIAL ACTION FOR GRAVITY WITH MATTER,
GAUGE-FIXING AND GHOSTS**

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ABSTRACT

Since the introduction of the Einstein-Hilbert variational principle for the equations of General Relativity, many alternative Lagrangians have been proposed, based on different choices of variables and with different algebraic or geometric structure. As long as an alternative Lagrangian can be reduced to the Einstein-Hilbert one by an algebraic redefinition or elimination of variables (that is, not involving the resolution of any differential equations), the alternative choice is largely equivalent to the original one. However, these alternative formulations may be useful for particular purposes, like development of geometric intuition, simplification of some calculations, deformation or coupling to other fields.

The Einstein-Hilbert Lagrangian (an n -form on an n -dimensional manifold) [6],

$$L_{\underline{a}}[g] = v_{\underline{a}}[g]g^{bc}R_{bc}[g], \quad (1)$$

is a function of the metric tensor g_{bc} , with g^{bc} the inverse metric, $v_{\underline{a}}[g]$ the metric volume form and $R_{bc}[g]$ the Ricci tensor. We are using a compressed index notation for n -forms, where $v_{\underline{a}} = v_{a_1 \dots a_n}$. When compared to other Lagrangians used in fundamental physics, like the scalar (Klein-Gordon), vector (Maxwell or Yang-Mills), spinor (Dirac), etc. Lagrangians, a striking distinction of the Einstein-Hilbert Lagrangian is that it is non-polynomial in the metric g_{bc} .

While not essential from the physical point of view, polynomiality is a convenient technical property. The main simplification it brings is at the level of perturbation theory, either classical or quantum. When writing the metric $g = \bar{g} + \varepsilon h_1 + \varepsilon^2 h_2 + \dots$ as an additive perturbation of a background metric \bar{g} , the equations for g give rise to a hierarchical family of linear equations for each of the h_i , with inhomogeneous sources depending polynomially on the $h_{j < i}$. For equations obtained from a non-polynomial Lagrangian like (1), at each order of perturbation theory, the order of the polynomial dependence of the inhomogeneous sources on the $h_{j < i}$ terms will increase. This polynomial order stays bounded only if the original Lagrangian was itself polynomial. In perturbative quantum field theory, we can analogously say that a non-polynomial Lagrangian gives rise to new and higher order *Feynman diagram vertices* [10] at each perturbative order. Thus, the polynomiality of the Lagrangian may significantly simplify the algebraic structure of perturbation theory, at least at higher orders.

A polynomial Lagrangian obviously gives rise to polynomial Euler-Lagrange equations. Another advantage of polynomiality shows itself at the level of the formal analysis of Euler-Lagrange equations as partial differential equations. Any partial differential equation can be geometrically and invariantly represented as an equation on the space of jets of the dynamical fields (the dependent variables) or, equivalently, the subspace of the jet space given by the vanishing locus of these equations [8]. When the equations are polynomial, this locus is an algebraic subvariety of the jet space, the *PDE subvariety*, while in general it is at best a smooth manifold or a stratified manifold, in the presence of singularities. Understanding the geometry of the PDE subvariety is crucial for identifying symmetries and conservation laws, identifying integrability conditions, prolonging the equations to involution, analyzing the singularities of solutions, etc. Thus, the powerful machinery of algebraic geometry, dedicated to the analysis of the geometric properties of algebraic varieties, may be brought to bear on these questions.

So, it is a natural question to ask whether General Relativity can be formulated with a polynomial Lagrangian. For pure vacuum General Relativity, the answer is Yes and the answer is provided by the cubic Einstein-Palatini formulation [3,7,2]

$$L_{\underline{a}}[\mathbf{g}, C] = \mathbf{g}_{\underline{a}}^{bc}(\bar{R}_{bc} + R_{bc}[C]), \quad \text{with} \quad R_{bc}[C] = -\bar{\nabla}_b C_{db}^d + \bar{\nabla}_d C_{bc}^d + C_{bc}^d C_d - C_{bd}^e C_{ce}^d, \quad (2)$$

where we should interpret the inverse densitized metric $\mathbf{g}_{\underline{a}}^{ab} = v_{\underline{a}}[g]g^{bc}$ as a fundamental variable, along with the Christoffel tensor C_{bc}^d , which parametrizes the difference between a connection ∇ and the Levi-Civita connection $\bar{\nabla}$ for a given background metric \bar{g} . Thus, \bar{R}_{bc} is the Ricci tensor of \bar{g} and $\bar{R}_{bc} + R_{bc}[C]$ becomes equal to the Ricci tensor for g , once C_{bc}^d assumes the standard Levi-Civita form for g . Since the variation with respect to C yields an equation for C equivalent to $\nabla_d g_{bc} = 0$, eliminating the auxiliary Christoffel tensor from the Lagrangian gives back the standard Einstein-Hilbert form, though written in terms of \mathbf{g} and thus commonly known as the Goldberg Lagrangian [4].

Unfortunately, it is not possible to maintain polynomial form with the same variables as above when expanding the Lagrangian by including a cosmological constant term or couplings to matter fields, both of which are of interest in physical applications. The challenge then becomes to introduce new auxiliary fields, like Lagrange multipliers, in terms of which a sufficiently rich Lagrangian may be written in polynomial form. We will content ourselves with adding (a) a cosmological constant, (b) a massive scalar field, (c) a (Maxwell or Yang-Mills) gauge field, (d) a Dirac spinor field, and (e) gauge-fixing and corresponding ghost terms for those fields that have gauge symmetries. Point (e) is a necessary ingredient for a consistent covariant quantization of this field theory [1]. Below, we give the final result, with some explanations:

The field content is $\Phi = (\mathbf{g}, C, B, u, \bar{u}, v, w, \phi, \hat{A}, \hat{F}, \hat{b}, \hat{z}, \hat{\bar{z}}, e, f, S, T, \psi)$, with its interpretation below. The hatted fields are Lie algebra valued. The Lie algebra is semi-simple, with commutator $[-, -]$ and invariant positive definite inner product $\langle -, - \rangle$. The real scalar and complex spinor multiplets carry representations of the Lie algebra, denoted $(-)\cdot\phi$ or $(-)\cdot\psi$, and invariant inner products $\langle -, - \rangle$, with $\langle \phi, A\cdot\phi \rangle = -\langle A\cdot\phi, \phi \rangle$ and $\langle \psi, A\cdot\psi \rangle = \langle A\cdot\psi, \psi \rangle$. We denote by $\bar{\gamma}^a$ a given choice of hermitian γ -matrices for the given background metric \bar{g}_{bc} , with $\bar{\nabla}$ its Levi-Civita connection, extended to spinors such that $\bar{\nabla}_b \bar{\gamma}^c = 0$.

\mathfrak{g}_a^{bc} : inverse densitized metric,	\hat{A}_b : gauge vector field,
$C_{(bc)}^a$: Christoffel tensor,	\hat{F}_b^c : gauge curvature tensor,
B_c : de Donder Nakanishi-Lautrup field,	\hat{b} : Lorenz Nakanishi-Lautrup field,
u^b, \bar{u}_c : diffeomorphism ghost and anti-ghost,	$\hat{z}, \hat{\bar{z}}$: gauge ghost and anti-ghost,
ϕ : scalar field (multiplet),	ψ : spinor (multiplet),
v_a : metric volume form,	$w^{\underline{k}^4 \dots \underline{k}^n}$: its Lagrange multiplier,
$e_{(bc)}$: Lorentz frame field,	$f^{(bc)}$: its Lagrange multiplier,
T_{bcd} : spin torsion,	S^{bcd} : its Lagrange multiplier.

$$\begin{aligned}
L_a[\Phi] = & \underbrace{\mathfrak{g}_a^{bc}(\bar{R}_{bc} + R_{bc}[C])}_{(a1):3} - \underbrace{B_b(\mathfrak{g}_a^{bc}B_c - 2\bar{\nabla}_c\mathfrak{g}_a^{bc})}_{(a2):3} - \underbrace{2\bar{\nabla}_{(b}\bar{u}_{c)}(\mathcal{L}_u\mathfrak{g}_a^{bc})}_{(a3):3} \\
& + \underbrace{w^{\underline{k}^4 \dots \underline{k}^n}(G_{\underline{a}\underline{k}^4 \dots \underline{k}^n}[\mathfrak{g}] + n!v_a v_{\underline{k}^4} \dots v_{\underline{k}^n})}_{(b0):n+1} - \underbrace{\frac{1}{2}(\mathfrak{g}_a^{bc}\langle(d\phi)_b, (d\phi)_c\rangle + m_\phi^2\langle\phi, \phi\rangle v_a)}_{(b1):3} \\
& + \underbrace{\frac{1}{4}\langle\hat{F}_b^c, v_a\hat{F}_c^b - 2\mathfrak{g}_a^{bd}(D\hat{A})_{cd}\rangle}_{(c1):3+1} + \underbrace{\frac{1}{2}\langle\hat{b}, v_a\hat{b} - 2\mathfrak{g}_a^{bc}\bar{\nabla}_b\hat{A}_c\rangle}_{(c2):3} + \underbrace{\frac{1}{2}\langle\bar{\nabla}_b(\mathfrak{g}_a^{bc}\hat{z}), (D\hat{z})_c\rangle}_{(c3):3+1} \\
& + \underbrace{f^{de}(\mathfrak{g}_a^{bc}e_{bd}e_{ce} - v_a\bar{g}_{de}) + S^{bcd}[v_a e_{cc'}\bar{g}^{c'e'}T_{bc'd} - (e_{fb}\bar{\nabla}_c e_{de})\mathfrak{g}_a^{ef}]}_{(d0):4} \\
& - \underbrace{\frac{1}{2}\mathfrak{g}_a^{bd}e_{bc}(\langle\psi, i\bar{\gamma}^c\bar{\nabla}_d\psi\rangle + \langle i\bar{\gamma}^c\bar{\nabla}_d\psi, \psi\rangle) - \frac{1}{4}v_a\langle\psi, i\bar{\gamma}^{[b}\bar{\gamma}^c\bar{\gamma}^{d]}\psi\rangle - m_\psi\langle\psi, \psi\rangle v_a}_{(d1):4} \\
& - \underbrace{2\Lambda v_a}_{(aa):1} - \underbrace{\lambda\phi^4 v_a}_{(bb):5} - \underbrace{\frac{m_A^2}{2}\mathfrak{g}_a^{bc}\langle\hat{A}_b, \hat{A}_c\rangle}_{(cc):3} + \underbrace{\frac{q_\phi}{2}\mathfrak{g}_a^{bc}(\langle\bar{\nabla}_c\phi, \hat{A}_b \cdot \phi\rangle + \langle\hat{A}_c \cdot \phi, \bar{\nabla}_b\phi\rangle) + \frac{q_\phi^2}{2}\mathfrak{g}_a^{bc}\langle\hat{A}_b \cdot \phi, \hat{A}_c \cdot \phi\rangle}_{(bc):5} \\
& - \underbrace{\mu\phi\langle\psi, \psi\rangle v_a}_{(bd):4} - \underbrace{\frac{q_\psi}{2}\mathfrak{g}_a^{bd}e_{bc}(\langle\psi, \bar{\gamma}^c\hat{A}_d \cdot \psi\rangle + \langle\bar{\gamma}^c\hat{A}_d \cdot \psi, \psi\rangle)}_{(cd):5}, \quad (3)
\end{aligned}$$

where the terms are labeled by their role and polynomial degree, and we have also used

$$R_{bc}[C] = -\bar{\nabla}_b C_{db}^d + \bar{\nabla}_d C_{bc}^d + C_{bc}^d C_d - C_{bd}^e C_{ce}^d, \quad (4)$$

$$(\mathcal{L}_u\mathfrak{g}_a^{bc}) = u^d\bar{\nabla}_d\mathfrak{g}_a^{bc} - 2\mathfrak{g}_a^{d(b}\bar{\nabla}_d u^{c)} + \mathfrak{g}_a^{bc}\bar{\nabla}_d u^d, \quad (5)$$

$$G_{\underline{a}\underline{k}^4 \dots \underline{k}^n}[\mathfrak{g}] = \mathfrak{g}_a^{b_1 c_1} \mathfrak{g}_b^{b_2 c_2} \mathfrak{g}_c^{b_3 c_3} \mathfrak{g}_{\underline{k}^4}^{b_4 c_4} \dots \mathfrak{g}_{\underline{k}^n}^{b_n c_n} \quad (\text{recalling } \underline{b} = b_1 \dots b_n, \underline{c} = c_1 \dots c_n), \quad (6)$$

$$(D\hat{A})_{dc} = (d\hat{A})_{dc} + [\hat{A}_d, \hat{A}_c], \quad (7)$$

$$(D\hat{z})_c = (d\hat{z})_c + [\hat{A}_c, \hat{z}]. \quad (8)$$

When \hat{A} is valued in an abelian Lie algebra, the polynomial degrees of (c1), (c3) drop by 1. The roles of the various terms are as follows.

(a1): gravity kinetic term,	(c1): gauge kinetic term,
(a2): de Donder gauge-fixing term,	(c2): Lorenz gauge-fixing term,
(a3): diffeomorphism ghost kinetic term,	(c3): gauge ghost kinetic term,
(b0): auxiliary fields for scalars,	(d0): auxiliary fields for spinors,
(b1): scalar kinetic term with mass,	(d1): spinor kinetic term with mass,
(aa): cosmological constant term,	(bc): scalar-gauge coupling,
(bb): scalar potential,	(bd): (Yukawa) scalar-spinor coupling,
(cc): (Proca) vector mass term,	(cd): gauge-spinor coupling.

It should be noted that the (Proca) vector mass term with $m_A^2 \neq 0$ is incompatible with the vector gauge invariance. So, for consistency, either only (cc) or only the (c2) and (c3) terms should be included. The scalar and spinor masses, m_ϕ^2 and m_ψ , the scalar coupling, λ , the scalar and spinor charges, q_ϕ and q_ψ , and the Yukawa coupling, μ , should be thought of as tensors with respect to the appropriate multiplet structures.

Upon eliminating all the auxiliary fields (those that can be eliminated by solving their Euler-Lagrange equations algebraically), the resulting Lagrangian is the standard Lagrangian for General Relativity coupled to a Standard Model-like matter theory. The overall polynomial degree of the Lagrangian is $\max\{5, n+1\}$, with the dimension dependent degree coming from the definition of the auxiliary volume form field, v_a . The above form is far from the only way of putting a similar Lagrangian in polynomial form (cf. [9], [5]). It is interesting to consider whether a polynomial form could be achieved with a smaller number of auxiliary fields or smaller polynomial degree in various terms.

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INTEGRAL GEOMETRY METHODS ON DERIVED CATEGORIES AND THEIR MODULI STACKS IN THE SPACE-TIME¹

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ABSTRACT

Geometrical moduli stacks are obtained through the Penrose transforms frame used on generalized \mathcal{D} -modules as $\mathcal{D}_{\mathbb{M}}$ -modules that are $\mathcal{D}_{\mathbb{P}}$ -modules (quasi-coherent \mathcal{D}_{λ} -modules with the corresponding character of a Hecke category [1], [2] which correspond to deformed images to some ramifications which can be classified by the theorem of correspondences to ramifications [3], where the unique geometrical pictures in field theory to different cohomological classes of the sheaves in category $\mathcal{D}^{\times}(\text{Bun}_G(\Sigma))$, are three type of fundamental co-cycles that are geometrical objects belonging to the global Langlands category (let monodromic or not) corresponding to a system $\text{Loc}_{L_G}(E^{\times})$, of the objects included in a category that involves a category of quasi-coherent sheaves on $\mathcal{D}G$, of certain fibers on the generalized flag manifolds that are λ -twisted D -modules of the flag variety G/B .

These quotients of cohomology classes on derived sheaves are generalized Verma modules [2] that in the ambit of solutions on the space-time of the field equations and using the Reillas's conjecture [2], [4] are classification spaces of $SO(1, n+1)$, where elements of the Lie algebra $\mathfrak{sl}(1, n+1)$ are differential operators, of the field equations in space-time [5], [6]. The cosmological problem that exists is to reduce the number of field equations that are resolvable under the same gauge field (*Verma modules*) and to extend the gauge solutions to other fields using the topological groups symmetries that define their interactions. Precisely the application of the Penrose transform ${}^L\text{Bun}_G$, let see which are generalized Verma modules of the Langlands geometrical equivalences $\mathcal{D}_{BRST}(\text{Oper}_{L_G}^{\leq n}(\mathcal{D}_y)) \cong \mathcal{D}^{\times}(\text{Bun}_G(\Sigma))$, which are demonstrated in [2]. But, the unique objects that are coefficients of the cohomological space of dimension 0, $(H^0(X, \mathcal{O}))$, [6] are the Verma modules $\mathbb{V}_{\lambda}(\chi), \forall \chi \in \text{Op}_{L_G}$. This extension can be given by a global Langlands correspondence between the Hecke sheaves category $\mathcal{H}_{G^{\wedge}, \infty}$, on an adequate moduli stack and the holomorphic L_G -bundles category with a special connection (*Deligne connection*). The corresponding $\mathcal{D}_{\mathbb{P}}$ -modules may be viewed as sheaves of conformal blocks (or co-invariants) (images under a generalized version of the Penrose transform of the type ${}^L\Phi^{\mu}(\mathcal{M}) = \mathcal{M} \boxtimes \rho^{\mu}(\mathbb{V})$) naturally arising in the framework

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of conformal field theory as isomorphisms between cohomological spaces of orbital spaces of the space-time.

Finally and searching to extend the structure of the holomorphic bundles to the meromorphic context, are obtained some moduli space identities [7] through line bundles of critical level and extend these to a line bundle $\tilde{\mathcal{L}}_\lambda^2$ to obtain cohomological classes solution of singular components of the Hitchin space-time (we consider the complex Riemannian manifold (non-compact manifold with singularities) as a Hitchin moduli space) using as basis a Higgs fields moduli space $\mathcal{M}_{Higgs}(G, C)$, to the obtaining meromorphic solutions to the corresponding enhanced part of the connection (singular part of the connection) or irregular ramification³). In this case the moduli space is the decomposition of the space-time as images in the dual by the Penrose transform which can be varieties where their moduli components can be super-projective spaces, for example $\mathcal{M}(0, \mathbb{P}^3) \cong \mathbb{P}^{3|4}$ [1], [7].

Following this order of these statements we can enunciate the following main result of this talk, considering that:

$$\mathcal{M}_{Higgs}(G, C) = T_V^\vee Bun_C(\Sigma), \tag{1}$$

is satisfied, where the Hitchin moduli space $\mathcal{M}_H(G, C)$, is interpreted as super-manifold of the space-time and each mapping is interpreted as the embedding of the string in space-time. Likewise, if we consider the vector bundle $P_C^{*V} \rightarrow C \times T_V^\vee Bun_C(\Sigma)$, that comes equipped with Higgs field $\phi \in H^0(C \times T_V^\vee Bun_C(\Sigma))$, characterized uniquely by the property that for every $\theta \in T_V^\vee Bun_C(\Sigma)$, we have $\phi|_{C \times \{\theta\}} = \theta$, since $\tilde{\mathcal{L}}_\lambda = \mathcal{L}_\lambda \otimes p^*V$ which comes given by the Penrose transform then we have the result:

Theorem (F. Bulnes) [7] Through to consider (1) and $\phi|_{C \times \{\theta\}} = \theta$, defined before, we have:

$$\mathcal{M}({}^L G, C) = \mathcal{M}_{Higgs}({}^L G, C) K^{1/2}, \tag{2}$$

where $K^{1/2}$, is the square root of the lines bundle on Bun_G , of the critical level.

²In $\tilde{\mathcal{L}}_\lambda = \mathcal{L}_\lambda \otimes p^*$. Here $\tilde{\partial} + \mathbb{Q}$, can be viewed as the connection of Deligne+other thing, [2] belonging to one "twisted" sub-category of \mathcal{D} - modules on the moduli stack with eigenvalues $E|_{X \setminus \{y_1, \dots, y_n\}}$ • Precisely $D_{coh}({}^L Bun, \mathcal{D}) \cong Ker(U, \tilde{\partial} + \nabla_s)$, then their images under the inverse Penrose transform are elements in sheaves of the category $\mathcal{D}_{BRST}(Oper_{L_G}^{\leq n})$, since by [1], [2] $\mathbb{Q}_{BRST}^2 = 0$, which is equivalent to the application of Cousin cohomology and their involved twistor transform has kernel isomorphic (this could be in $Z(\mathfrak{g}_{\mathcal{K}_c})$) to $FunOp_L(\mathcal{D}^\times)$. By the *Opers* theory $Op_L(\mathcal{D}^\times) \cong proj(\mathcal{D}^\times) \times \oplus \Omega_{\mathcal{K}}^{\otimes(d_j+1)}$, where $\Omega_{\mathcal{K}}^{\otimes n}$, is the space of n -differentials on \mathcal{D}^\times , and $proj(\mathcal{D}^\times)$, is the $\Omega_{\mathcal{K}}^{\otimes 2}$ - torsor of projective connections on \mathcal{D}^\times , which is conformed ∇_s . complex variable.

³This is much seemed to the analytic continuation studied in complex variable. In this case is more complicated, since $\nabla+$ *ramifications*, can be viewed as images under functors of the type $\Phi+$ *Geometrical hypothesis*, using our integral transforms.

Proof sketch: We consider our space-time $\mathbb{M} = \mathcal{M}_H(G, C)$, as a complex Riemannian manifold with the symplectic structure for ω_k . Then over $F = \omega_J = \frac{2}{e^2} \int_C \text{Tr} \delta\phi \wedge * \delta A$, $\forall A$, a gauge field, we have $e^2 F = \bar{F}$, which is a flat connection component to F .

For other way, K is a canonical line bundle of Bun_G . Furthermore $\mathcal{M}_H(G, C) \cong \mathcal{M}_{Higgs}(G, C)$, where holomorphic corresponding coordinates are the gauge field pairs (A, ϕ) .

Applying the twistor transform to relate the cohomological decomposing between lines bundles, we have $\mathcal{T} : H_{[\mathbb{L}]}^d(X, \tilde{\mathcal{L}}) \longrightarrow \cong H^{d-1}(X - \tilde{\mathbb{L}}, \tilde{\mathcal{L}})$, where between isomorphisms of Harish-Chandra modules there is a factor whose co-dimension is given by $\dim \mathcal{M}(G, C)$,⁴ then

$$\bar{K}^2 \mathcal{M}(G, C) = \mathcal{M}_{Higgs}(G, C) K, \quad (3)$$

but $\bar{K} = I_F K$, and $\mathcal{M}_H(G, C) \rightarrow Bun_G(C) |_K$, then

$$I_F K \mathcal{M}(G, C) = \mathcal{M}_{Higgs}(G, C) K^{1/2}. \quad (4)$$

But $I_F K \mathcal{M}(G, C)$ is a decomposing or factorization of the moduli space $\mathcal{M}_H(G, C)$ (which is our non-compact manifold with singularities) where is the decomposition of the space-time as images in the dual by Penrose transform which can be varieties where their moduli components can be super-projective spaces, for example $\mathcal{M}(\mathbb{P}^3, 0) \cong \mathbb{P}^{3|4}$, as has been mentioned before. Using the L -oper, in the second member of (4) we have the identity. \square

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⁴Remember that the space $\mathcal{M}_H(G, C)$, is a non-compact manifold of dimension $4(g-1)\dim G$, with singularities.

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Keywords: Geometrical Langlands correspondence, Hecke categories, moduli stacks, Penrose transforms, quasi-coherent sheaves, twisted \mathcal{D} - modules sheaves, generalized Verma modules.

DIFFERENTIAL INVARIANTS**Zdeněk Dušek**

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ABSTRACT

A geometric approach to the theory of differential invariants will be presented. The categories of fibre bundles, the relation between the differential invariants and certain classes of morphisms of fibre bundles, the structure of the differential group and the factorization method with respect to a distinguished subgroup will be recalled, following [1] and [2]. A result on characterization of differential invariants of the metric tensor by [3] will be also recalled. As a continuation, differential invariants of the second order of the metric tensor and a 1-form will be described.

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MSC2010: 53A55, 58A20

Keywords: Jet of a diffeomorphism, r-frame bundle, differential group, differential invariant.

EXOTICA OR THE FAILURE OF THE STRONG COSMIC CENSORSHIP IN FOUR DIMENSIONS

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ABSTRACT

Based on [1] in this talk a generic counterexample to the strong cosmic censor conjecture is exhibited. More precisely—taking into account that the conjecture lacks any precise formulation yet—first we make sense of what one would mean by a "generic counterexample" by introducing the mathematically unambiguous and logically stronger concept of a "robust counterexample". Then making use of Penrose' nonlinear graviton construction (i.e., twistor theory) and a Wick rotation trick we construct a smooth Ricci-flat but not flat Lorentzian metric on the largest member of the Gompf–Taubes uncountable radial family of large exotic \mathbb{R}^4 's. We observe that this solution of the Lorentzian vacuum Einstein's equations with vanishing cosmological constant provides us with a sort of counterexample which is weaker than a "robust counterexample" but still reasonable to consider as a "generic counterexample". It is interesting that this kind of counterexample exists only in four dimensions. Motivated by Gompf's *exotic ménagerie* construction we also speculate that the existence of this counterexample may reflect the general situation in four dimensions.

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**HOLONOMY THEORY AND RECURRENCE OF TENSORS ON
4-DIMENSIONAL MANIFOLDS**

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ABSTRACT

Let M be a 4-dimensional, smooth, connected manifold admitting a smooth metric g of arbitrary signature and with associated Levi-Civita connection ∇ . Suppose T is a global, smooth tensor field on M . Then T will be called *recurrent* if

$$\nabla T = T \otimes P$$

for some smooth 1-form P on M . In studying a recurrent tensor field on M , it will be assumed that it is nowhere-zero on the non-empty, connected, open subset U on which it is studied. Suppose that T is recurrent (and nowhere-zero) on the non-empty, open, connected subset U of M . If P vanishes on U , T is *parallel* (covariantly constant) on U .

Let T be a second order symmetric tensor at $m \in M$ with components T_{ab} . One can associate with T a linear map $f : T_m M \rightarrow T_m M$ on $T_m M$, the tangent space to M at $m \in M$, by $u^a \rightarrow T^a_b u^b$ and consider its (real or complex) eigenvectors and eigenvalues represented, respectively, by k and α (with respect to the metric g) and given by

$$T^a_b k^b = \alpha k^a \quad (T_{ab} k^b = \alpha g_{ab} k^b).$$

One may classify such a tensor by finding all possible Jordan forms and Segre types for f . Such a classification for the metric of signature $(+, +, -, -)$ (neutral signature) has been given (see, e.g. [6]) and the possible canonical forms and Segre types are listed. This classification is similar to the corresponding one for Lorentz signature and which is used in general relativity theory [1] but is more complicated owing to the existence of independent pairs of orthogonal null vectors and pairs of orthogonal timelike 2-spaces in neutral signature.

The purpose of this study is to examine the properties of recurrence for vector fields and second order symmetric tensor fields on a 4-dimensional manifold M admitting a metric g whose signature (up to sign) is one of the (only) possibilities $(+, +, +, +)$ (positive definite signature), $(+, +, +, -)$ (Lorentz signature) and $(+, +, -, -)$ (neutral signature). The general idea is first to solve the problem of parallel (or scalable so as to be parallel) vector and second order symmetric tensor fields for (M, g) and then to extend this idea to the recurrent tensor fields which are not in this category and which will be called *properly recurrent*. The largest

part of this study is to solve the general problem for recurrent second order symmetric tensors on a 4-dimensional manifold M admitting a neutral signature metric. The techniques used are based on the classification of second order symmetric tensors and holonomy theory the latter using the classification of the subalgebras of $o(2, 2)$ given in [3] (see also [9]). These results are then applied to the Ricci tensor and the problem of Ricci recurrence is solved for a non-flat manifold admitting a metric g of signature $(+, +, -, -)$. In this application, direct algebraic methods and the Ambrose-Singer theorem [5] (see also [4]) are used and the holonomy types which allow Einstein spaces are obtained by this procedure. The general recurrence problem for second order symmetric tensors for Lorentz signature has already been given [2] but will here be done using the holonomy techniques in [8], [1] for this signature and some examples will be given. A similar review, using the holonomy techniques in [7], will also be given when g has positive definite signature.

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COVARIANT DIFFERENTIAL IDENTITIES AND CONSERVATION LAWS IN METRIC-TORSION THEORIES OF GRAVITATION

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ABSTRACT

General manifestly generally covariant formalism for constructing conservation laws and conserved quantities in arbitrary metric-torsion theories of gravitation, which recently has been elaborated by the authors [1–5] is presented. It is assumed that Lagrangians \mathcal{L} of such a theories depend on metric tensor \mathbf{g} , curvature tensor \mathbf{R} , torsion tensor \mathbf{T} and its first $\nabla\mathbf{T}$ and second $\nabla\nabla\mathbf{T}$ covariant derivatives, besides, on an arbitrary set of other tensor (matter) fields φ and their first $\nabla\varphi$ and second $\nabla\nabla\varphi$ covariant derivatives:

$$\mathcal{L} = \mathcal{L}(\mathbf{g}, \mathbf{R}; \mathbf{T}, \nabla\mathbf{T}, \nabla\nabla\mathbf{T}; \varphi, \nabla\varphi, \nabla\nabla\varphi).$$

Thus, both the standard minimal coupling with the Riemann-Cartan geometry and non-minimal coupling with the curvature and torsion tensors are considered.

The studies and results are as follow. (a) A physical interpretation of the Noether and Klein identities is examined. It was found that they are the basis for constructing equations of balance of energy-momentum tensors of various types (canonical, metrical and Belinfante symmetrized). The equations of balance are presented. (b) Using the generalized equations of balance, new (generalized) manifestly generally covariant expressions for canonical energy-momentum and spin tensors of the matter fields are constructed. In the cases, when the matter Lagrangian contains both the higher derivatives and non-minimal coupling with curvature and torsion, such generalizations are non-trivial. (c) The Belinfante procedure is generalized for an arbitrary Riemann-Cartan space. (d) A more convenient in applications generalized expression for the canonical superpotential is obtained. (e) A total system of equations for the gravitational fields and matter sources are presented in the form more naturally generalizing the Einstein-Cartan equations with matter. This result, being a one of more important results itself, is to be also a basis for constructing physically sensible conservation laws and their applications.

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Keywords: diffeomorphic invariance, manifest covariance, differential identities, conservation laws, stress-energy-momentum tensors, spin tensors, metric-torsion theories, gravity, Riemann-Cartan geometry.

BEYOND EINSTEIN GRAVITY: THE GEOMETRIC DEFORMATION

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ABSTRACT

General Relativity (GR), in its hundred years of existence, has proved to be a successful and well tested theory for gravitation. However there are some important issues associated to the gravitational interaction which GR cannot answer satisfactorily, mainly, 1) The inability of GR to explain the dark matter and dark energy problem without the use of some kind of unknown matter-energy to reconcile what predicts GR with the observed, namely, galactic rotation curves and accelerated expansion of the universe. 2) The impossibility to reconcile GR with the Standard Model of particle physics, leading thus to the inability to quantize gravity. All this has motivated the searching of a new gravitational theory beyond GR that helps to explain part of the problems mentioned above. Indeed, there are many alternative theories, as for instance, high curvature gravity theories, Galileon theories, $f(R)$ gravity theories, scalar-tensor theories, massive gravity, new massive gravity, topologically massive Gravity, Chern-Simons theories, higher spin gravity theories, Horava-Lifshitz gravity, extra-dimensional theories, etc. However, the quantum gravity is still an open problem, and the dark matter and dark energy remain a mystery so far.

Any extension to GR will eventually produce new terms in the Einstein equations. These “corrections” are usually handled as part of an effective energy-momentum tensor and appear in such a way that they should vanish or be negligible in an appropriate limit, as for instance, at solar system scale, where GR has been successfully tested. This limit represents not only a critical point when a consistent extension to GR is studied, but also a non-trivial problem that must be treated carefully.

The simplest way to produce an extension to GR is by considering a modification on Hilbert-Einstein action as

$$S = \frac{1}{2k^2} \int R \sqrt{-g} d^4x + \alpha (\text{correction}) , \quad (1)$$

where α is a free parameter of the “superior” theory which drives the corrections to GR. The explicit form corresponding to the generic correction shown in Eq. (1) should be, of course, a well justified and physically motivated expression. At this stage the GR limit, obtained by imposing $\alpha = 0$, is just a trivial issue, so everything looks consistent so far. Indeed, we may

go further and calculate the equations of motion corresponding to this new theory, then as usual we demand $\delta S = 0$ to obtain

$$k^2 T_{\mu\nu} + \alpha(\text{new terms})_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} , \quad (2)$$

where the new terms in Eq. (2) may be consolidated as part of an effective energy-momentum tensor. The explicit form of this tensor may contain some new fields, as for instance scalar fields, vector fields and tensor fields. All of them coming from the new gravitational sector not described by Einstein's theory. At this stage the GR limit, again, is a trivial issue, since $\alpha = 0$ leads to the standard Einstein's equations $k^2 T_{\mu\nu} = G_{\mu\nu}$.

All above seems to tell us that the consistent problem, namely the GR limit, is a trivial issue. However, when the system of equations given by the expression (2) is solved, the result shows a complete different story. In general, and this is very common, the solution eventually found cannot reproduce the GR limit by imposing $\alpha = 0$. The cause of this problem is the high non-linearity of the system shown generically in Eq. (2), so this problem should not be a surprise. To clarify this point, let us consider a spherically symmetric perfect fluid, whose solution for the g_{rr} metric component is given by

$$g_{rr}^{-1} = 1 - \frac{2m(r)}{r} , \quad (3)$$

where $m(r)$ is the mass function of the self-gravitating system. Now let us consider the same perfect fluid under the "new" gravitational theory shown in Eq. (1). When the Eq. (2) is solved we obtain an expression which generically may be written as

$$g_{rr}^{-1} = 1 - \frac{2m(r)}{r} + (\text{geometric deformation}) , \quad (4)$$

where the *geometric deformation* in Eq. (4) should be understood as the deformation undergone by the metric component in Eq. (3) due to the generic extension of GR shown in Eq. (1). Therefore "deformation" means a deformation from a GR point of view. Now, and this is very important, the deformation shown in the expression (4) always produce *anisotropic consequences* on the perfect fluid. Hence, the self-gravitating system is not a perfect fluid anymore. Indeed, and this is a critical point in our analysis, the anisotropy \mathcal{P} produced by the geometric deformation always appears as

$$\mathcal{P} = A + \alpha B . \quad (5)$$

The expression in Eq. (5) is very significant, since it shows that the GR limit cannot be regained. When $\alpha = 0$ there is a "sector" in the anisotropy shown in Eq. (5) which remains, this is the α -independent sector which is generically called A in Eq. (5). In consequence, the GR solution, namely the perfect fluid solution, is not contained in this extension. Therefore we have an extension to GR which does not contain GR. This is of course a contradiction, or more properly a consistence problem. The source of this problem comes from the *geometric deformation* shown in Eq. (4). This deformation always appear as

$$\text{geometric deformation} = X + \alpha Y , \quad (6)$$

that is, there is a “sector” of the geometric deformation which is α -independent, generically called X in Eq. (6). Obviously this is a contradiction, since the deformation undergone by GR must be α -dependent. The source of this problem, again, is the high non-linearity of the effective Einstein equations shown in Eq. (2). We want to emphasise that this problem has nothing to do with any specific extension of GR. Indeed, it is a characteristic of high non-linear systems.

A method to solve the non-trivial issue describe above is the so-called *Minimal Geometric Deformation Approach* MGD [1,2]. The idea is to keep under control the anisotropic consequences on GR coming from the extended theory, in such a way that the α -independent sector in the geometric deformation shown in Eq. (6), namely X , vanishes. In consequence A , which represents the α -independent sector of the anisotropy in Eq. (5), also will vanish. This ensures a consistent extension having GR in the limit $\alpha = 0$. In this approach the generic expression Y in Eq. (6) represents the *minimal geometric deformation* undergone by the radial metric component, being the generic expression B in Eq. (5) the *minimal anisotropic consequence* undergone by GR due to the correction terms in the Hilbert-Einstein action shown in Eq. (1).

How to implement the approach describe above? Or more clearly, how to make $X = 0$ in Eq. (6) to obtain a consistent extension to GR? This is accomplished when a GR solution is forced to keep being a solution in the extended theory. We need to introduce the GR solution into the new theory, as far as possible. This provides the foundation for the MGD approach [3-12]. We want to emphasise that the GR solution used to makes $X = 0$ in Eq. (6) will eventually be modified by using, for instance, the matching conditions at the surface of a self-gravitating system, leading thus to physical variables as functions of the free parameter of the theory, here generically named α . This free parameter represents, for instance, the parameter to measure deviation from GR in $f(R)$ theories, the brane tension in the braneworld, and so.

Finally we want to mention that, in general, the deformation undergone for the radial metric component should be considered along with a deformation in the temporal metric component. Indeed, it was proved in the braneworld context, that a temporal deformation due to extra-dimensional effects produces part of the deformation in the radial metric component [13]. Also it was found, in the specific case of $f(R)$ theories, that the deformation in the radial metric component must be considered along with the temporal deformation. Otherwise the inconsistency associated with the GR limit cannot be removed in $f(R)$ theories.

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ON THE METRIZABILITY PROBLEM FOR AFFINE CONNECTIONS

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ABSTRACT

The local metrizable problem for symmetric affine connections on manifolds has been tackled by many authors, applying different methods and obtaining merely some necessary and sufficient conditions for existence of its solutions. However, the explicit formulas of metrizable connections and the corresponding metric fields are usually not found. Recently, the metrizable problem has been considered for affine connections which are invariant with respect to Lie groups (see [4]), and in particular cases of symmetries the explicit solutions of the metrizable problem were found. In this work we follow a direct approach to the problem as initiated in [1], and obtain necessary and sufficient conditions for solvability of the metrizable equation, based on complete integrability criterion (Frobenius theorem) for systems of first order partial differential equations. Applying the inverse variational problem approach to the metrizable problem (cf. [2], [3]), we describe metric fields compatible with a given connection in an implicit form.

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Keywords: Affine connection, metrizable, complete integrability, inverse variational problem.

GENERALIZED HAMILTONIAN GRAVITY

10-plectic formulation of 4-bein gravity

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ABSTRACT

We present a multisymplectic formulation for the first order vierbein gravity, which is as much covariant as possible¹, i.e. which does not depend on choices of space-time coordinates nor on the trivialization of the principal bundle. The multisymplectic formulation of the Einstein-Cartan action of vierbein gravity raises difficulties because one of the dynamical fields is the Lorentz connection and is subject to gauge invariance. In addition, other difficulties appear when we take into account the solder form, which is the other dynamical field necessary for the description of the first order theory.

Dynamical fields in the Einstein-Cartan formulation can be defined locally as being pairs (e, A) , where e is a moving coframe on the space-time \mathcal{X} (defining a metric on the tangent bundle $T\mathcal{X}$ by requiring e to be dual to a Lorentzian orthonormal frame) and A is a \mathfrak{g} -valued connection 1-form on \mathcal{X} . The Einstein-Cartan action functional (which is also termed the Palatini action functional of vierbein (tetrad) gravity) then reads $\mathcal{A}_{\text{EC}}[A, e] = \int_{\mathcal{X}} \frac{1}{2} \varepsilon_{abc}{}^d e^a \wedge e^b \wedge (dA + A \wedge A)^c{}_d$, where $F := dA + A \wedge A$ is the curvature form, $F^{cd} := \mathfrak{h}^{dd'} F^c{}_{d'}$. It is possible to understand pairs (e, A) by assuming that a rank 4 vector bundle \mathcal{V} has been chosen over \mathcal{X} , equipped with a pseudo-metric \mathfrak{h} . Then A represents a connection of \mathcal{V} which respects the pseudo-metric \mathfrak{h} and e represents a *solder form*, i.e. a rank 4 section of the vector bundle over \mathcal{X} whose fiber over $x \in \mathcal{X}$ is the set of linear maps from $T_x\mathcal{X}$ to \mathcal{V}_x .

It is well-known that the Einstein-Cartan action is invariant by gauge transformations of the form $(e, A) \mapsto (g^{-1}e, g^{-1}dg + g^{-1}Ag)$. One way to picture geometrically this ambiguity is to lift the variational problem on the *total space* \mathcal{P} of the principal bundle $(\mathcal{P}, \mathcal{X}, \pi, \mathfrak{G})$ of orthonormal frames on the space-time manifold \mathcal{X} , with a right action of $\mathfrak{G} := \text{SO}(3, 1)$. This is the *internal* vector bundle $(\mathcal{V}, \mathcal{X}, \pi)$, where the typical fiber is the Minkowski vector space $\vec{\mathbb{M}}$. We denote by $\mathfrak{p} := \mathfrak{g} \oplus \mathfrak{t}$ the Poincaré Lie algebra of the Poincaré Lie group $\mathfrak{P} = \mathfrak{G} \ltimes \mathfrak{T}$ (the group of affine isometries of \mathbb{M} , where $(\mathbb{M}, \mathfrak{h})$ is the Minkowski affine space), \mathfrak{g} is the Lorentz Lie algebra and \mathfrak{t} is the trivial Lie algebra of \mathfrak{T} , the Abelian Lie group of translations on the Minkowski space.

Following ideas which are now quite standard since Élie Cartan, we lift the Cartan connection defined on the space-time (e, A) to the principal bundle \mathcal{P} over \mathcal{X} . The connection

¹The ideas and motives presented in [1] are developed in the context of the first order Einstein-Cartan formulation of vierbein gravity [2, 3].

is then represented by a \mathfrak{p} -valued 1-form $\boldsymbol{\eta} = \overset{\circ}{\boldsymbol{\eta}} + \overset{\hat{1}}{\boldsymbol{\eta}} \in \Omega^1(\mathcal{P}, \mathfrak{p})$, The 1-form $\boldsymbol{\eta}$ is defined on the total space of the principal fiber bundle \mathcal{P} , it satisfies a *normalization* and an *equivariance* hypothesis. Although *a priori* necessary the equivariance condition has the drawback of being a *non-holonomic* constraint on the first order derivatives of the field. Within the multisymplectic framework we realize that it is not necessary to assume the equivariance condition *a priori*, since this condition is a consequence of the dynamical equations.

We work on the De Donder-Weyl m -plectic multimomentum bundle $\Lambda_1^m T^*(\mathfrak{p} \otimes T^*\mathcal{P})$ ($m = \dim(\mathcal{P}) = 10$), which is constructed on the space of \mathfrak{p} -valued 1-form on \mathcal{P} , denoted $\mathfrak{p} \otimes T^*\mathcal{P}$. The Legendre transform for the Einstein-Cartan action is computed by treating connections as normalized and equivariant \mathfrak{p} -valued 1-forms on \mathcal{P} . We find that the natural multisymplectic manifold can be built from the vector bundles $\mathfrak{p} \otimes T^*\mathcal{P}$ and $\mathfrak{p}^* \otimes \Lambda^8 T^*\mathcal{P}$ over \mathcal{P} , where \mathfrak{p}^* its dual of \mathfrak{p} . These vector bundles are endowed with a canonical \mathfrak{p} -valued 1-form $\boldsymbol{\eta}$ and a canonical \mathfrak{p}^* -valued 8-form p respectively. Inside $\mathfrak{p} \otimes T^*\mathcal{P}$ we consider the subbundle $\mathfrak{p} \otimes^N T^*\mathcal{P}$ of *normalized* forms. Then the multisymplectic manifold corresponds to the total space of the vector bundle $\mathcal{M} := \mathbb{R} \oplus_{\mathcal{P}} (\mathfrak{p} \otimes^N T^*\mathcal{P}) \oplus_{\mathcal{P}} (\mathfrak{p}^* \otimes \Lambda^8 T^*\mathcal{P})$, equipped with the 10-form $\theta = \varsigma \beta \wedge \gamma + p \wedge (d\boldsymbol{\eta} + \boldsymbol{\eta} \wedge \boldsymbol{\eta})$ where ς a coordinate on \mathbb{R} , $\beta \wedge \gamma$ is the volume form on \mathcal{P} , and $p := -\frac{1}{2} p^{\mu\nu} \beta_{\mu\nu} \wedge \gamma + p^{\mu j} \beta_{\mu} \wedge \gamma_j$. Note that (dx^{μ}, γ^i) is a co-frame on \mathcal{P} , where $\mu = 0, \dots, 3$ and $i = 1, \dots, 6 = \dim(\mathfrak{G})$. We denote $\beta := dx^0 \wedge \dots \wedge dx^3$ and $\gamma := \gamma^1 \wedge \dots \wedge \gamma^6$. Also we denote the contractions $\beta_{\mu_1 \dots \mu_p} := (\partial_{\mu_1} \wedge \dots \wedge \partial_{\mu_p}) \lrcorner \beta$, and $\gamma_{i_1 \dots i_p} := (\rho_{i_1} \wedge \dots \wedge \rho_{i_p}) \lrcorner \gamma$, where (∂_{μ}, ρ_i) is a moving frame on \mathcal{P} . In this setting we discover that we may *remove* the unnatural equivariance constraint and derive the corresponding generalized Hamilton equations without this assumption.

The solutions of the Hamilton-Volterra-De Donder-Weyl (HVDW) equations are given by 10-dimensional submanifold of \mathcal{M} , (any solutions as graph can be represented as the image of an unique embedding $\mathcal{P} \ni z \mapsto (z, \boldsymbol{\kappa}(z), \boldsymbol{\eta}(z), \boldsymbol{p}(z)) \in \mathcal{M}$. In particular, we denote by $\boldsymbol{\eta} \in \Gamma(\mathcal{P}, \mathfrak{p} \otimes^N T^*\mathcal{P})$ a section such that $\boldsymbol{\eta}(z) = \boldsymbol{\eta}$ (viewing $\boldsymbol{\eta}$ as a map from \mathcal{P} to the total space of the bundle $\mathfrak{p} \otimes^N T^*\mathcal{P}$). We introduce:

$$\begin{aligned} \overset{\circ}{\widehat{\mathbf{G}}}_c^d &= \varepsilon_{abc}{}^d (d\overset{\hat{1}}{\boldsymbol{\eta}}^a \wedge \overset{\hat{1}}{\boldsymbol{\eta}}^b + \overset{\circ}{\boldsymbol{\eta}}_a^a \wedge \overset{\hat{1}}{\boldsymbol{\eta}}^{a'} \wedge \overset{\hat{1}}{\boldsymbol{\eta}}^b), & \overset{\circ}{\widehat{\boldsymbol{\Lambda}}}_c^{d\mu} &= \overset{\circ}{\boldsymbol{p}}_{c;j}^{d\mu j} + [\mathbf{l}_j, \overset{\circ}{\boldsymbol{p}}^{\mu j}]_c^d, \\ \overset{\hat{1}}{\widehat{\mathbf{G}}}_a &= \varepsilon_{abc}{}^d (d\overset{\circ}{\boldsymbol{\eta}}^a + \overset{\circ}{\boldsymbol{\eta}}^a \wedge \overset{\hat{1}}{\boldsymbol{\eta}}_d^c \wedge \overset{\hat{1}}{\boldsymbol{\eta}}^b), & \overset{\hat{1}}{\widehat{\boldsymbol{\Lambda}}}_a^{\mu} &= \overset{\hat{1}}{\boldsymbol{p}}_{a;j}^{\mu j} - \overset{\hat{1}}{\boldsymbol{p}}_b^{\mu j} \mathbf{l}_{j a}^b, \end{aligned}$$

where $\overset{\circ}{\widehat{\mathbf{G}}}_c^d := (1/3!) \varepsilon^{\sigma\lambda\mu\nu} \overset{\circ}{\widehat{\mathbf{G}}}_{c\lambda\mu\nu}^d \beta_{\sigma}$ (torsion 3-form), $\overset{\hat{1}}{\widehat{\mathbf{G}}}_a := (1/3!) \varepsilon^{\sigma\lambda\mu\nu} \overset{\hat{1}}{\widehat{\mathbf{G}}}_{a\lambda\mu\nu} \beta_{\sigma}$ (Einstein 3-form), $\overset{\circ}{\widehat{\boldsymbol{\Lambda}}}_c^{d\mu}$ and $\overset{\hat{1}}{\widehat{\boldsymbol{\Lambda}}}_a^{\mu}$ are written by using the standard matrix representation of \mathfrak{p} . The basis of the Poincaré algebra is written $(t_a, u_{ib}^a)_{1 \leq i \leq 6}$, where $t_a \in \mathfrak{t}$ and each basis element $u_i \in \mathfrak{g}$ is given by the matrix element u_{ib}^a , we also note $u_i^{ab} := u_{ib'}^a \mathbf{h}^{b'b}$ and remark that $u_i^{ab} + u_i^{ba} = 0$. The HVDW yields some interesting generalization of the Einstein-Cartan system of equations:

$$\begin{aligned} (1/3!) \varepsilon^{\sigma\lambda\mu\nu} \overset{\circ}{\widehat{\mathbf{G}}}_{a\lambda\mu\nu}^b &= \overset{\circ}{\widehat{\boldsymbol{\Lambda}}}_a^{b\sigma} \\ (1/3!) \varepsilon^{\sigma\lambda\mu\nu} \overset{\hat{1}}{\widehat{\mathbf{G}}}_{a\lambda\mu\nu} &= \overset{\hat{1}}{\widehat{\boldsymbol{\Lambda}}}_a^{\sigma} \end{aligned} \tag{1}$$

The HVDW equations yields the existence of e_{μ} and A_{μ} , i.e. some \mathfrak{t} -valued and \mathfrak{g} -valued functions, respectively, which depend only on x (and not on g) such that $\forall x \in \mathcal{X}, \forall g \in \mathfrak{G}$,

$\overset{\circ}{\eta}_\mu := g^{-1}A_\mu(x)g$ and $\overset{1}{\eta}_\mu = g^{-1}e_\mu(x)$, respectively. Therefore, we obtain *dynamically* the local equivariance properties of a Cartan connection. In addition, we set $\overset{1}{\mathbf{G}}_a := \varepsilon_{abc}{}^d(dA + A \wedge A)^c \wedge e^b$, $\overset{\circ}{\mathbf{G}}_c^d := \varepsilon_{abc}{}^d(de^a + A_{a'}^a \wedge e)^{a'} \wedge e^b$ and we also introduce the gravitational charges $\overset{1}{\mathbf{\Lambda}}_a^{\mu j} := \overset{1}{\mathbf{p}}_{a'}^{\mu j}(g^{-1})_{a'}^{a'}$ and $\overset{\circ}{\mathbf{\Lambda}}_c^{d\mu j} := (g)_{d'}^d \overset{\circ}{\mathbf{p}}_{c'}^{d'\mu j}(g^{-1})_{c'}^{c'}$. Then, the HVDW equations reduce to:

$$\begin{aligned} (1/3!)\varepsilon^{\sigma\lambda\mu\nu}\overset{\circ}{\mathbf{G}}_{c\lambda\mu\nu}^d &= \overset{\circ}{\mathbf{\Lambda}}_{c;j}^{d\sigma j} \\ (1/3!)\varepsilon^{\sigma\lambda\mu\nu}\overset{1}{\mathbf{G}}_{a\lambda\mu\nu} &= \overset{1}{\mathbf{\Lambda}}_{a;j}^{\sigma j} \end{aligned} \quad (2)$$

The additional difficulties raised by the non-compactness of the Lorentz group are rooted in the right hand side of equations (2). As opposed to the case of the Yang-Mills system [1], the gravitational charges $\overset{1}{\mathbf{\Lambda}}_a^{\mu j}$ and $\overset{\circ}{\mathbf{\Lambda}}_c^{d\mu j}$ are delicate to deal with. Nonetheless, the underlying architecture of the HVDW system of equations might shed new light on unknown territories e.g. in Cartan geometry, higher gauge theory, cosmology, and quantum gravity.

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ON SCHMIDT'S B-BOUNDARY IN THE CATEGORY OF DIFFERENTIAL SPACES

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ABSTRACT

A spacetime is a four dimensional smooth manifold equipped with the Lorentzian metric. Orthonormal frames over a spacetime constitute a fibre bundle. The metric connection on a spacetime induces a parallelization of a connected component of this fibre bundle. This parallelization induces the Riemannian metric on this component. Therefore, the Cauchy boundary of this component can be considered. The component itself can be identified with the base manifold (a spacetime), whereas the Cauchy boundary with some kind of a boundary of a spacetime.

This construction, proposed by Schmidt, can be equivalently expressed in terms of spectra, i.e., real homomorphisms of a certain algebra of functions. Spectras can be seen as differential spaces. It seems to be quite helpful and useful. For example, a certain classification of b-boundaries can be done, based on the properties of differential spaces and the impact of a superposition closure and localization procedures on the initial algebra of functions generating the given differential spaces. Indeed, the language of differential spaces (a generalization of a smooth manifold) occurs to be very useful in the problem of singular spacetimes.

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NONLINEARLY CHARGED BLACK HOLES IN GRAVITY'S RAINBOW

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ABSTRACT

Motivated by interesting results of gravity's rainbow and also nonlinear electrodynamics, we consider charged rainbow solutions and investigate their thermodynamic properties. We take into account a model of nonlinear electrodynamics and study their effects on the gravity's rainbow spacetime.

1. Introduction and Motivations

One of the weighty dream of physicists is creating a consistent quantum theory of gravity. Although various aspects of the quantum gravity is still very much alive, but till now, all attempts of obtaining a complete description of the quantum gravity are unfeasible. Some of theoreticians believe that the violation of Lorentz invariancy is an essential rule to construct quantum theory of gravity. The Lorentz invariance violation may be expressed in form of modified dispersion relations [1-5]. In other words, according to the loop quantum gravity (LQG) results [2] and spacetime discreteness assumption [5] at the Planck scale, one may regard Lorentz invariance violation or modified dispersion relation by redefining the physical momentum and energy. One of the mentioned modified dispersion relation, which is predicted by new approach of quantum gravity [6,7], takes the following general form

$$f^2(E/E_p)E^2 - g^2(E/E_p)p^2 = m^2, \quad (1)$$

where E_p is the Planck energy, and the temporal and spatial rainbow's functions ($f(E/E_p)$ and $g(E/E_p)$) satisfy the following conditions

$$\lim_{E/E_p \rightarrow 0} f(E/E_p) = 1, \quad \lim_{E/E_p \rightarrow 0} g(E/E_p) = 1. \quad (2)$$

This modified dispersion relation can be originated from the so-called doubly special relativity [8]. Doubly special relativity is a natural extension of special relativity, which enjoys the invariancy of the speed of light, to the case of assuming the invariancy of the energy scale (the Planck energy) [9]. Including the curvature into the Doubly special relativity, Magueijo and Smolin proposed the so-called doubly general relativity [10]. In other words, doubly general relativity is a natural extension of doubly special relativity to the case of assuming

curved spacetime. Regarding the doubly general relativity, one finds the geometry of spacetime may depend on the energy of the particle moving in it (E). In other words, spacetime is parameterized by the ratio $E = E_p$ to obtain a parametric family of metrics, the so-called rainbow of metrics [10]. Hence, the modified metric in gravity's rainbow can be written as [10]

$$g(E) = \eta^{ab} e_a(E) \otimes e_b(E), \quad (3)$$

in which the energy dependence of the frame fields is

$$e_0(E) = \frac{1}{f(E/E_P)} \hat{e}_0, \quad e_i(E) = \frac{1}{g(E/E_P)} \hat{e}_i, \quad (4)$$

where the hatted quantities refer to the energy independent frame fields. The functional form of rainbow functions is based on various phenomenological motivations.

2. Basic Field Equations and solutions

The Lagrangian of Einstein gravity with cosmological constant coupled to an electromagnetic field may be written as The Lagrangian of Einstein gravity with cosmological constant coupled to an electromagnetic field may be written as

$$L_{\text{tot}} = R - 2\Lambda - \mathcal{L}(\mathcal{F}), \quad (5)$$

where R and Λ are, respectively, the Ricci scalar and the cosmological constant, and electromagnetic Lagrangian, $\mathcal{L}(\mathcal{F})$, is a function of Maxwell invariant $\mathcal{F} = F_{ab}F^{ab}$, where the Faraday tensor is $F_{ab} = \partial_{[a}A_{b]}$ and A_b is the gauge potential. Taking into account the gauge-gravity Lagrangian (5) and applying the variational method, we obtain

$$G_{ab} + \Lambda g_{ab} = \frac{1}{2} g_{\mu\nu} L(\mathcal{F}) - 2F_{\mu\lambda} F_{\nu}^{\lambda} L_{\mathcal{F}}, \quad (6)$$

$$\nabla_a (\mathcal{L}_{\mathcal{F}} F^{ab}) = 0, \quad (7)$$

where G_{ab} is the Einstein tensor, $\mathcal{L}_{\mathcal{F}} = \frac{d\mathcal{L}(\mathcal{F})}{d\mathcal{F}}$.

Here, we consider the Born-Infeld nonlinear electrodynamics [11] as a matter field and obtain the modified spacetime with regarding gravity's rainbow. To do so, we consider the following metric

$$ds^2 = -\frac{\Psi(r)}{f(E)^2} dt^2 + \frac{1}{g(E)^2} \left(\frac{dr^2}{\Psi(r)} + r^2 d\phi^2 \right). \quad (8)$$

Taking into account metric (8) with the field equations (6) and (7), one can find that

$$F_{tr} = \frac{q}{r\Gamma} \quad (9)$$

$$\Psi(r) = -\frac{\Lambda r^2}{g(E)^2} - m + \frac{2r^2\beta^2(1-\Gamma)}{g(E)^2} + q^2 f(E)^2 - 2q^2 f(E)^2 \ln \left(\frac{r(1+\Gamma)}{2l} \right), \quad (10)$$

where $\Gamma = \sqrt{1 + \frac{f(E)^2 g(E)^2 q^2}{r^2 \beta^2}}$, and two integration constants q and m are, respectively, related to the electric charge and mass of the black hole.

Calculation of scalar curvatures shows that the mentioned metric can be interpreted as black hole with physical temperature. In addition, one can find that obtained conserved and thermodynamic quantities satisfy the first law of thermodynamics. Moreover, it may be shown that obtained black hole is thermally stable. In other words, although gravity's rainbow functions modified various properties of the solutions, they did not affect stability conditions (see [12] for more details). Since it was shown that gravity's rainbow leads to the existence of remnants for black holes, it will be interesting to discuss remnants of conserved and thermodynamic quantities as a consequence of gravity's rainbow.

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A BRIEF LOOK AT DIFFERENTIAL INVARIANTS OF THE METRIC TENSOR

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ABSTRACT

Theory of differential invariants is very extensive, there are several methods for finding differential invariants. In this paper we present a method, which was at first used by Krupka [3]. This method is based on properties of jet structures which are factorized by differential group L_n^r or some of its normal subgroups. For illustration we remind the results concerning differential invariants of the metric tensor [5]. These results obtained by this factorization method are in correspondence with classical approach used in theoretical physical papers. General theory can be found in [1], [4].

Let $\text{Met } X$ denotes the space of metrics on a smooth manifold X , let $n = \dim X$. Then $\text{Met } X$ has a structure of a fiber bundle with type fiber of regular symmetric second order covariant tensors $\mathbb{R}^{n*} \odot \mathbb{R}^{n*}$, associated with bundle of frames over X . Let L_n^r be the r -th differential group of invertible r -jets with source and target at $0 \in \mathbb{R}^n$. Denote by $T_n^r(\mathbb{R}^{n*} \odot \mathbb{R}^{n*})$ the prolongation of the left L_n^1 -manifold $\mathbb{R}^{n*} \odot \mathbb{R}^{n*}$, i.e. the set of r -jets with source at $0 \in \mathbb{R}^n$ and target in the $\mathbb{R}^{n*} \odot \mathbb{R}^{n*}$ with natural structure of the left L_n^{r+1} -manifold (see [2]). The canonical coordinates $(g_{ij}, g_{ij,k_1}, g_{ij,k_1k_2}, \dots, g_{ij,k_1k_2\dots k_r})$ can be replaced by adapted coordinates which are subset of functions $g_{ij}, \Gamma_{i,j_1j_2}, \Gamma_{i,j_1j_2j_3}, \dots, \Gamma_{i,j_1j_2j_3\dots j_{r+1}}, R_{ijkl}, R_{ijkl;m_1}, \dots, R_{ijkl;m_1;\dots;m_{r-2}}$, where $\Gamma_{i,j_1j_2j_3\dots j_s}$ are symmetric in indices $j_1, j_2, j_3, \dots, j_s$ and $R_{ijkl;m_1;\dots;m_s}$ denotes the s -th covariant derivative of curvature tensor R_{ijkl} . In the case $r = 2$ we denote adapted coordinates by $(g_{ij}, \Gamma_{i,jk}, R_{ijkl}, \Gamma_{i,jkl})$. Transformation formulas are given by

$$\begin{aligned} g_{ij} &= g_{ij}, \\ \Gamma_{i,jk} &= \frac{1}{2}(g_{ij,k} + g_{ik,j} - g_{jk,i}), \\ R_{ijkl} &= \frac{1}{2}(g_{il,jk} + g_{jk,il} - g_{ik,jl} - g_{jl,ik}) \\ &\quad + \frac{1}{4}g^{mp}((g_{mj,k} + g_{mk,j} - g_{jk,m})(g_{pi,l} + g_{pl,i} - g_{il,p}) \\ &\quad - (g_{mj,l} + g_{ml,j} - g_{jl,m})(g_{pi,k} + g_{pk,i} - g_{ik,p})), \\ \Gamma_{i,jkl} &= \frac{1}{3}(g_{ij,kl} + g_{il,jk} + g_{ik,lj}) - \frac{1}{6}(g_{jk,li} + g_{lj,ki} + g_{kl,ji}). \end{aligned}$$

Let K_n^{r+1} be the kernel of the canonical projection homomorphism $\pi_n^{r+1,1} : L_n^{r+1} \rightarrow L_n^1$. We will be interested in the quotient space of the action of K_n^{r+1} on the $T_n^r(\mathbb{R}^{n*} \odot \mathbb{R}^{n*})$. For the coordinate expressions of this action we refer to [5], this action is free.

Let $A \in L_n^3$, $A = J_0^3 \alpha$, be an invertible 3-jet with source and target at $0 \in \mathbb{R}^n$. Let $(b_j^i, b_{jk}^i, b_{jkl}^i)$, $1 \leq i \leq n$, $1 \leq j \leq k \leq l \leq n$, be coordinate system on L_n^3 defined by

$$b_j^i(A) = D_j \alpha_i^{-1}(0), \quad b_{jk}^i(A) = D_j D_k \alpha_i^{-1}(0), \quad b_{jkl}^i(A) = D_j D_k D_l \alpha_i^{-1}(0).$$

Let $Q \in T_n^2(\mathbb{R}^{n*} \odot \mathbb{R}^{n*})$, $Q = J_0^2 f$, be a 2-jet with source at $0 \in \mathbb{R}^n$ and target at $\mathbb{R}^{n*} \odot \mathbb{R}^{n*}$. Denote

$$\begin{aligned} g_{ij} &= g_{ij}(Q), & \Gamma_{i,jk} &= \Gamma_{i,jk}(Q), & R_{ijkl} &= R_{ijkl}(Q), & \Gamma_{i,jkl} &= \Gamma_{i,jkl}(Q), \\ \bar{g}_{ij} &= g_{ij}(A \cdot Q), & \bar{\Gamma}_{i,jk} &= \Gamma_{i,jk}(A \cdot Q), & \bar{R}_{ijkl} &= R_{ijkl}(A \cdot Q), & \bar{\Gamma}_{i,jkl} &= \Gamma_{i,jkl}(A \cdot Q). \end{aligned}$$

Then the action of subgroup K_n^3 on $T_n^2(\mathbb{R}^{n*} \odot \mathbb{R}^{n*})$ is given in adapted coordinates by

$$\begin{aligned} \bar{g}_{ij} &= g_{ij}, \\ \bar{\Gamma}_{i,jk} &= \Gamma_{i,jk} + b_{jk}^p g_{ip}, \\ \bar{R}_{ijkl} &= R_{ijkl}, \\ \bar{\Gamma}_{i,jkl} &= \Gamma_{i,jkl} + b_{kl}^p \Gamma_{i,jp} + b_{lj}^p \Gamma_{i,kp} + b_{kj}^p \Gamma_{i,lp} \\ &\quad + \frac{1}{3}(b_{kl}^p \Gamma_{p,ij} + b_{lj}^p \Gamma_{p,ik} + b_{kj}^p \Gamma_{p,il}) \\ &\quad + \frac{1}{3}(b_{ki}^p \Gamma_{p,jl} + b_{ji}^p \Gamma_{p,kl} + b_{li}^p \Gamma_{p,jk}) \\ &\quad + \frac{1}{3}(b_{ki}^p b_{jl}^q + b_{li}^p b_{jk}^q + b_{ji}^p b_{lk}^q) g_{pq} + b_{jkl}^p g_{ip}. \end{aligned}$$

Let Q be an arbitrary L_n^1 -manifold. The mapping

$$F : T_n^r(\mathbb{R}^{n*} \odot \mathbb{R}^{n*}) \rightarrow Q$$

is called *the r -th order differential invariant of the metric tensor*, if it satisfies the condition

$$F(A \cdot J_0^r f) = \pi^{r+1,1}(A) \cdot F(J_0^r f)$$

for each $J_0^r f \in T_n^r(\mathbb{R}^{n*} \odot \mathbb{R}^{n*})$, $A \in L_n^r$.

Let P_n be the subspace of the tensor space $(\mathbb{R}^{n*} \wedge \mathbb{R}^{n*}) \odot (\mathbb{R}^{n*} \wedge \mathbb{R}^{n*})$ which is in canonical coordinates R_{ijkl} determined by the condition

$$R_{ijkl} + R_{iljk} + R_{iklj} = 0.$$

Dimension of P_n is the same as the number of coordinate functions R_{ijkl} on $T_n^2(\mathbb{R}^{n*} \odot \mathbb{R}^{n*})$ in adapted coordinate system $(g_{ij}, \Gamma_{i,jk}, R_{ijkl}, \Gamma_{i,jkl})$ and is equal to $(1/12)n^2(n^2 - 1)$.

The L_n^3 -manifold $T_n^2(\mathbb{R}^{n*} \odot \mathbb{R}^{n*})$ has the structure of a left principal K_n^3 -bundle. This left principal K_n^3 -bundle is trivial, and its base $T_n^2(\mathbb{R}^{n*} \odot \mathbb{R}^{n*})/K_n^3$ is diffeomorphic to the $(\mathbb{R}^{n*} \odot \mathbb{R}^{n*}) \times P_n$.

Theorem 1 *Every differential invariant from the left L_n^3 -manifold $T_n^2(\mathbb{R}^{n*} \odot \mathbb{R}^{n*})$ to any left L_n^1 -manifold Q depends only on g_{ij} and R_{ijkl} .*

Similarly we can prove that the L_n^{r+1} -manifold $T_n^r(\mathbb{R}^{n*} \odot \mathbb{R}^{n*})$ has the structure of the left principal K_n^{r+1} -bundle. This left principal K_n^{r+1} -bundle is trivial, and its base is diffeomorphic to some Euclidean space.

Theorem 2 *Every differential invariant from the left L_n^{r+1} -manifold $T_n^r(\mathbb{R}^{n*} \odot \mathbb{R}^{n*})$ to any left L_n^1 -manifold Q depends only on g_{ij} and R_{ijkl} , $R_{ijkl;m}$, $R_{ijkl;m_1;m_2}$, \dots , $R_{ijkl;m_1;\dots;m_{r-2}}$.*

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